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# ON THE FINE STRUCTURE IN ALPHA DECAY

BY

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The coupling scheme associated with large nuclear deformations, and appropriate to the very heavy nuclei, suggests that the preferred mode of  $\alpha$ -decay involves nucleons in paired orbits. For the even-even nuclei, the strongest transition thus goes to the ground state of the daughter, and the fine structure populates predominantly the rotational band associated with this configuration; for odd-A nuclei, the favoured  $\alpha$ -decays populate the rotational band for which the orbit of the last odd nucleon remains unchanged. From this interpretation one also obtains a number of correlations between energies and lifetimes of the favoured  $\alpha$ -decays in even-even and odd-A nuclei. Moreover, it is possible to estimate the intensity ratios in the favoured  $\alpha$ -decays of odd-A nuclei on the basis of those observed in the even-even nuclei.

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**M**any striking regularities have been observed in the  $\alpha$ -decay fine structure of the heavy elements, especially for the even-even nuclei, and it has been shown that these regularities are related to the fact that the  $\alpha$ -decay process primarily populates states in the daughter belonging to a rotational band (ASARO and PERLMAN, 1953, 1954; FALK-VAIRANT, TEILLAC, VALLADAS, and BENOIST, 1954; MILSTED, ROSENBLUM, and VALADARES, 1954; cf. also the review article by NEWTON, 1954).

In the present paper, we shall consider a simple picture of the  $\alpha$ -particle formation, related to the coupling scheme of these strongly deformed nuclei, which makes possible a further discussion of the regularities in the  $\alpha$  fine structure. This interpretation also implies a number of relationships between the  $\alpha$ -decay of even-even and odd-A nuclei.

In nuclei whose equilibrium shape possesses a large deformation, one expects a band of rotational states associated with each configuration of the particles<sup>1</sup>. In most cases, the nuclear

<sup>1</sup> For a more detailed discussion of nuclear rotational states, cf. BOHR and MOTTELSON, 1953, 1954; and A. BOHR, 1954.

shape is expected to possess axial symmetry, and the rotational spectrum is then given by

$$E = \frac{\hbar^2}{2\mathfrak{J}} \{I(I+1) + a(-1)^{I+1/2}(I+1/2)\delta_{K,1/2}\} \quad (1)$$

apart from a constant depending on the intrinsic nuclear structure. The quantum number  $K$  represents the projection of the total angular momentum along the nuclear symmetry axis and is constant for all the states in a given rotational band. The effective moment of inertia which depends on the nuclear deformation is denoted by  $\mathfrak{J}$ , and the quantity  $a$  is a decoupling parameter which occurs only in the rotational spectra for configurations with  $K = 1/2$ .

The independent particle motion in axially symmetric deformed potentials may be characterized by the quantum number  $\Omega_p$  representing the projection of the total angular momentum of the particle along the nuclear symmetry axis. States differing only in the sign of  $\Omega_p$  are degenerate, and the lowest state of a nucleus is thus obtained by filling the particles pairwise in states of opposite  $\Omega_p$ .

In an even-even nucleus, the ground state configuration therefore has  $K = (\sum_p \Omega_p) = 0$ , and the associated rotational band contains the levels

$$I = 0, 2, 4, 6 \dots \text{(even parity)}. \quad (2)$$

In an odd-A nucleus,  $K$  is equal to  $|\Omega_p|$  for the last odd nucleon, and for each binding state of this particle there is a rotational band containing the states

$$I = K, K+1, K+2, \dots \left. \begin{array}{l} \text{(all same parity as} \\ \text{the orbit of the last} \\ \text{odd nucleon).} \end{array} \right\} \quad (3)$$

Particles occupying states which differ only in the sign of  $\Omega_p$  interact especially strongly due to the large overlap of their wave functions. Thus, the formation of an  $\alpha$ -particle from an independent particle system of this structure is expected to take place most easily from two such pairs of protons and neutrons. This type of  $\alpha$ -decay, which is characterized by the selection

rules  $\Delta K = 0$  and no change of parity, will be referred to as the favoured  $\alpha$ -transitions.

A further consequence of the non-spherical field of the nucleus is the exchange of angular momentum between  $\alpha$ -particle and daughter nucleus, which implies that in general several

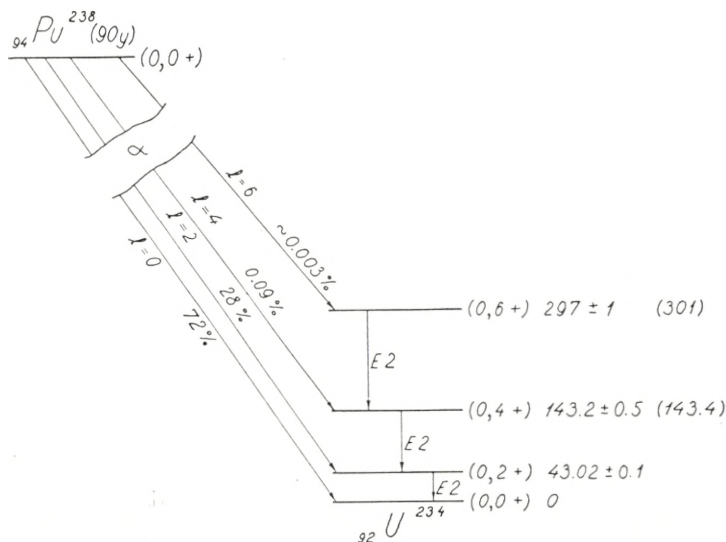


Fig. 1. *Alpha-decay of Pu<sup>238</sup>*. The experimental data is taken from NEWTON and ROSE, 1953, and from ASARO and PERLMAN, 1954 a. The states are labeled by the quantum numbers  $(K, I\pi)$ , where  $\pi$  is the parity.

The  $\alpha$ -decay of  $\text{Pu}^{238}$ , which illustrates the typical pattern observed in the even-even  $\alpha$ -emitters with  $A > 220$ , populates the rotational sequence associated with the ground state configuration  $(0, I+)$  of the daughter nucleus. The observed energies are given in keV; the values shown in parenthesis are calculated from (1) by adjusting the moment of inertia to give the observed energy of the first excited state. The small deviations between the calculated and measured values have a negative sign and increase with  $I$ , as is expected for the correction terms to (1) resulting from the rotation-vibration interaction (cf. BOHR and MOTTELSON, 1953, 1954). This correction has the form  $-R I^2 (I + 1)^2$ . The value of  $R$ , estimated from the observed energies, is  $R \approx 2 \times 10^{-3}$  keV, which is of the expected order of magnitude.

members of each rotational band are populated with comparable intensity.

In an even-even nucleus, the  $\alpha$ -decay should thus take place primarily to the lowest  $K = 0$  band of the daughter, i. e. the ground state and its rotational excitations. This pattern has indeed been observed as a systematic feature of the  $\alpha$  fine structure in even-even nuclei possessing large deformations ( $A > 220$ )

(ASARO and PERLMAN, 1953; NEWTON, 1954); for an example, cf. Fig. 1. The spins and energies of the populated excited states are in agreement with (2) and (1). The rotational character of the excitations is further supported by the  $E2$  transition probabilities which are several orders of magnitude greater than for single-particle transitions (cf. NEWTON, 1954; TEMMER and HEYDENBURG, 1954).

The great similarity in the decay process for the different even-even nuclei in this region is exhibited by the marked regularity in the lifetime energy relations for the ground state transitions (PERLMAN, GHIORSO, and SEABORG, 1950; KAPLAN, 1951). With relatively good accuracy (within about a factor of two), the transition probability per second,  $P_0$ , for all these transitions can be represented by the simple Geiger-Nuttal law

$$\log_{10} P_0 = C - \frac{D}{\sqrt{E}}, \quad (4)$$

where  $E$  is the kinetic energy of the  $\alpha$ -particle, while  $C$  and  $D$  are constants for each element and vary regularly with  $Z$ . The relationship (4) as well as the order of magnitude of  $C$  and  $D$  can be obtained from the theory of barrier penetration (cf., e. g., GAMOW and CRITCHFIELD, 1949). The values of  $C$  and  $D$ , determined by a recent analysis (FRÖMAN, 1955) of the empirical data, are listed in Table I.

TABLE I.

$Z$	$C$	$D$
84 . . . . .	50.15	128.8
86 . . . . .	50.94	132.7
88 . . . . .	51.51	136.2
90 . . . . .	51.94	139.4
92 . . . . .	52.55	143.1
94 . . . . .	53.35	147.4
96 . . . . .	53.97	151.3
98 . . . . .	54.40	154.7

The table lists the coefficients appearing in the empirical Geiger-Nuttal relation (Eq. (4)) for even-even ground state transitions. The units employed are such that, when the  $\alpha$ -energy is measured in MeV, the transition probability is given in  $\text{sec}^{-1}$ . The coefficients are listed as functions of the charge number  $Z$  of the parent nucleus.

There is also a considerable regularity in the intensities of the  $\alpha$  fine structure, leading to the rotational excitations of the ground state configurations. The observed transition probabilities to the  $(2+)$  levels are of the order of (4), while those to the  $(4+)$  and  $(6+)$  levels are systematically smaller (ASARO and PERLMAN, 1953). This hindrance, which shows a regular variation with  $A$ , amounting in the heaviest elements to a factor of about a hundred, can be ascribed only partly to the centrifugal barrier encountered by the  $\alpha$ -particle.

A detailed theory of the intensity rules for the population of the rotational band in the daughter nucleus would involve, in the first place, a consideration of the  $\alpha$  formation process and of the boundary conditions it implies at the non-spherical nuclear surface. In the second place, there would be involved a treatment of the penetration problem for the  $\alpha$ -wave through the anisotropic Coulomb barrier (HILL and WHEELER, 1953). Due to the exchange of angular momentum between the  $\alpha$ -particle and the residual nucleus, this latter part of the  $\alpha$  decay process is described by a system of coupled differential equations (RASMUSSEN, 1954a; FRÖMAN, 1955).

The problem is simplified, however, by the fact that the formation of the  $\alpha$ -particle and its passage through the region close to the nuclear surface, in which appreciable exchange of angular momentum may occur, takes place in a time short compared to the nuclear rotational period. It is, therefore, possible to a first approximation to consider the nucleus as fixed in space during the  $\alpha$ -particles' traversal of this region<sup>1</sup>. We shall later exploit this simplification in the comparison between the  $\alpha$ -decay of even-even and odd- $A$  nuclei.

The  $\alpha$  fine structure pattern in even-even nuclei so far discussed has included only the favoured transitions in which the  $\alpha$ -particle is formed from nucleons in paired orbits, and thus leaves the daughter even-even nucleus in the ground state rotational band<sup>2</sup>. It is of course also possible for the  $\alpha$ -decay to populate

<sup>1</sup> For a more detailed discussion of this approximation and of the solutions obtained, cf. FRÖMAN, 1955.

<sup>2</sup> Favoured transitions may also be expected to take place to excited configurations with  $K = 0$ , representing either the excitation of a pair of particles or a collective vibration. Such states may, however, have a rather high excitation energy ( $\sim 1$  MeV) and, in fact, such transitions seem not so far to have been observed.

excited particle configurations in the daughter nucleus; examples of such unfavoured transitions have been observed in the  $\alpha$ -spectrum of  $\text{Th}^{230}$  ( $\alpha_{255}$ ) (BOUSSIERES et al., 1953; RASETTI and BOOTH, 1953; VALLADAS and BERNAS, 1953) and  $\text{Th}^{228}$  ( $\alpha_{217}$ ) (BOUSSIERES et al., 1953; ASARO, STEPHENS, and PERLMAN,

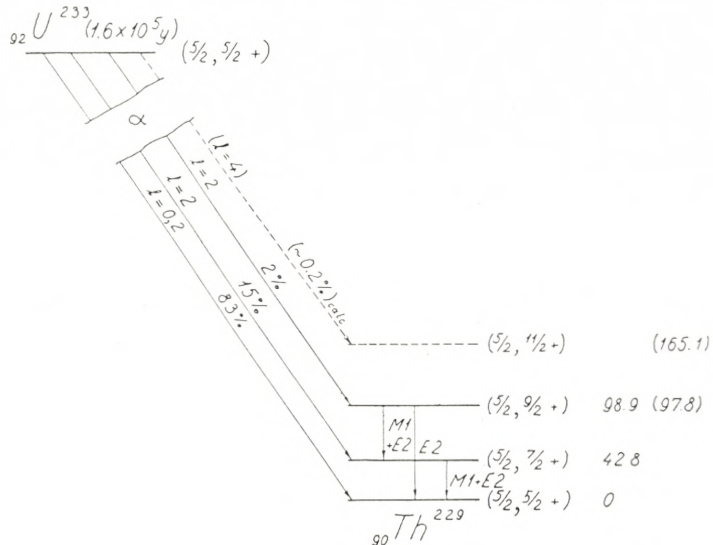


Fig. 2. *Alpha-decay of  $\text{U}^{233}$ .* The decay scheme is based on experimental data given in the references found in Table III. The notation is the same as in Fig. 1.

The  $\alpha$ -decay is interpreted as taking place to the ground state rotational band in  $\text{Th}^{229}$ . The value of  $K$  for this band is deduced, by means of (1) and (3), from the measured energies of the first and second excited states, and is found to equal the known spin of the parent nucleus, as is expected for the favoured  $\alpha$ -transitions. The interpretation of the transitions as of the favoured type is further supported by the reduced transition probability for the ground state transition (cf. Table II). For a theoretical estimate of the relative intensities of the fine structure components, cf. Table IV.

The position of the expected, but so far not observed, third excited state is deduced from (1), and the intensity of the  $\alpha$ -group populating this state is calculated from (5) (cf. Table IV). The parities in the figure represent values relative to that of the  $\text{U}^{233}$  ground state, and are based on the interpretation of the  $\alpha$ -transitions as favoured, which implies no change of parity.

1953; NEWTON and ROSE, 1954). As expected, the transition probabilities are appreciably smaller than for the favoured transitions given by (4).<sup>1</sup>

<sup>1</sup> Note added in proof: Recently, similar transitions have been identified in the  $\alpha$ -spectra of other neighbouring nuclei, and evidence has been provided for the (1-) character of the excited states in question (STEPHENS, ASARO, and PERLMAN, 1954). It has been suggested (R. F. CHRISTY, private communication) that these excitations may be of collective type associated with a nuclear shape containing deformations of odd multipole type.



In an odd-A nucleus, the favoured  $\alpha$ -decays leave the last odd particle moving in the same orbital in the daughter as in the parent<sup>1</sup>. If this configuration is that of the ground state band of the daughter nucleus, the expected fine structure pattern is much the same as in even-even nuclei with the most intense transition going to the ground state and regularly decreasing intensities of the branchings to the rotational excitations of this configuration. An example of such a decay is provided by  $U^{233}$  (cf. Fig. 2).

If the orbital of the last odd particle in the parent corresponds to an excited particle configuration in the daughter the transitions to the ground state band are hindered and the decay is expected to take place predominantly to the rotational band associated with the excited configuration in question (provided its excitation energy is not too great).

As an example of such a pattern in an odd-A nucleus, the  $\alpha$ -decay of  $Am^{241}$  is shown in Fig. 3 (RASMUSSEN, 1954; ASARO and PERLMAN, 1954; MILSTED, ROSENBLUM, and VALADARES, 1954).

If the favoured transitions leave the daughter with appreciable excitation energy ( $\gtrsim 1/2$  MeV), these transitions, although intrinsically preferred, may be relatively weak. In this case, the  $\alpha$  fine structure may be somewhat more complex with no single transitions dominating. The  $\alpha$ -decays of  $Th^{227}$  and  $Pa^{231}$  appear to provide examples of this type (FRILLAY, ROSENBLUM, VALADARES, and BOUSSIERES, 1954; ROSENBLUM, COTTON, and BOUSSIERES, 1949).

The similarity between the favoured  $\alpha$ -transitions in odd-A and even-even nuclei implies a number of simple relations between the two classes of transitions, as regards energy systematics, lifetimes, and fine structure intensities.

In particular, a close correspondence is expected between the even-even ground state transitions and the favoured odd-A transitions to the head of the daughter band (i. e. having  $\Delta I = 0$ ). The decay energies of the two types of transitions should exhibit similar trends, and the odd-A transitions in question should also follow approximately the same Geiger-Nuttal law (4) as the even-even ground state transitions.

<sup>1</sup> A similar interpretation of the  $\alpha$  fine structure in odd-A nuclei has recently been discussed by NEWTON, 1954, and by RASMUSSEN, private communication.

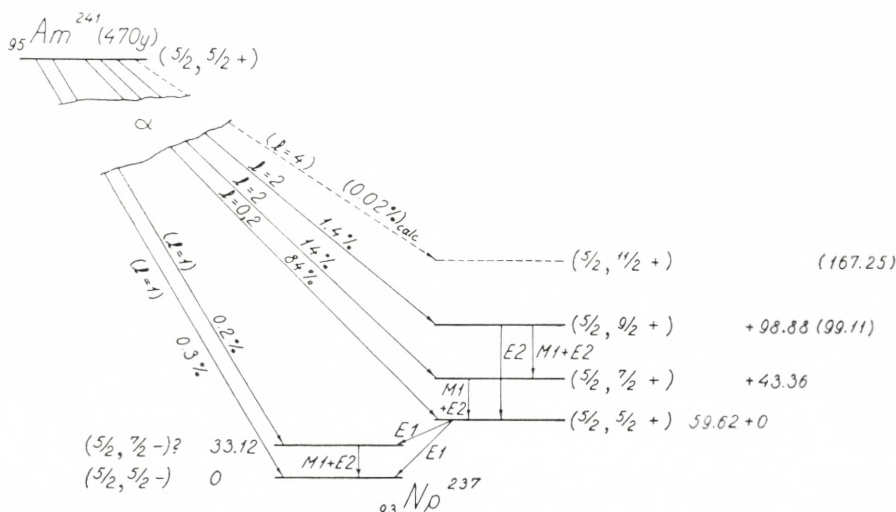


Fig. 3. Alpha-decay of  $Am^{241}$ . The decay scheme is based on experimental data given in the references found in Table III. The states are labeled as in Fig. 1. Parities are relative to that of the  $Am^{241}$  ground state. Levels which are interpreted as belonging to the same rotational band in  $Np^{237}$  are drawn above each other, while the two different rotational bands are displaced sideways.

The favoured  $\alpha$ -transitions take place to the 60 keV level and its rotational excitations. The accurately measured energies in this series show the  $K$ -value to be  $5/2$ . The very small difference between the observed value for the  $(5/2, 9/2+)$  level and the value calculated from (1) (shown in parenthesis) is just of the sign and magnitude expected from the rotation-vibration interaction, as observed in  $U^{234}$  (cf. Fig. 1). (For  $Np^{237}$ , one finds  $R \approx 1.5 \times 10^{-3}$  keV, which agrees with the value obtained from  $U^{234}$ , considering the expected variation of  $R$  with  $\left(\frac{h^2}{2\mathcal{I}}\right)^3$  (cf. BOHR and MOTTELSON, 1953, p. 91)). The  $\alpha$ -intensity to the expected  $(5/2, 11/2+)$  level has been estimated from (5) (cf. Table IV).

From (5), one also estimates that the  $\alpha$ -transition to the  $(5/2, 5/2+)$  level should be 80%  $l = 0$  and 20%  $l = 2$ . Evidence on this mixture may be obtained from the  $\alpha$ - $\gamma$  correlation between this  $\alpha$ -group and the 60 keV E1  $\gamma$ -ray. From the calculated mixture and the spin values given in the figure, one estimates

$$W(\theta) = 1 \pm .35 P_2(\cos \theta). \text{ An angular anisotropy of } A = \frac{W(\pi)}{W(\pi/2)} - 1 = -0.21$$

has been found (FRASER and MILTON, 1954) to decay with a lifetime of  $5.5 \times 10^{-9}$  sec. However, as evidenced by the  $\alpha$ - $\gamma$  correlations in even-even nuclei, one expects for these strongly deformed nuclei, attenuations from quadrupole couplings acting over times appreciably shorter than those studied. The observed anisotropy which is about half that estimated therefore appears consistent with the present interpretation of the decay.

The  $\alpha$ -decay to the ground state of  $Np^{237}$  is strongly hindered ( $F \sim 10^{-3}$ ). If one tentatively assumes the 33 keV level to be a rotational excitation of the ground state, and the  $\alpha$ -decays to both these states to be predominantly  $l = 1$ , one estimates from (5) the relative intensities 1:0.25, in favour of the ground state transition. This interpretation of the first excited state would also imply a branching ratio of 1:0.035 for the E1  $\gamma$ -rays from the  $(5/2, 5/2+)$  level to the  $(5/2, 5/2-)$  and  $(5/2, 7/2-)$  levels (cf. ALAGA, ALDER, BOHR, and MOTTELSON, 1955). The observed ratio is 1:0.07 (BELING, NEWTON, and ROSE, 1952).

Thus, the observed transition probabilities may be used to identify the favoured odd-A transitions in a given decay scheme. The  $\alpha$ -transitions in odd-A nuclei, tentatively classified in this manner as favoured and having  $\Delta I = 0$ , are listed in Table II, together with the even-even ground state transitions.

The table gives the  $\alpha$ -energy and reduced transition probability  $F$ , which is the ratio of the observed transition probability to that given by (4). Although the classification of the odd-A transitions is in some cases rather uncertain, there are seen to exist for almost all the odd-A  $\alpha$ -emitters groups with transition probabilities comparable to those of the even-even ground state transitions<sup>1</sup>. The fluctuation in  $F$  for the odd-A and even-even transitions seem to be comparable, apart from a few uncertain cases, but there appears to be a systematic tendency for the odd-A  $F$ -values to be somewhat smaller than unity, on the average by a factor of about two. As will be seen below, there are a number of such small systematic differences between the odd-A and even-even favoured transitions.

The  $\alpha$ -energies of the favoured  $\Delta I = 0$  transitions are plotted in Fig. 4. It is seen that the odd-A and even-even transitions exhibit closely parallel trends with, however, a systematic tendency for the odd-A energies to be smaller, by about 200 keV, than those interpolated between the even-even nuclei.

Such an effect may be understood in terms of the change of the kinetic energy of the last odd particle associated with the small shrinking of the nuclear volume. Thus, if one assumes the nuclear radius to be proportional to  $A^{1/3}$ , the difference in radius between the parent and daughter nuclei implies an energy shift of just a few hundred keV.

In cases where a rotational fine structure has been observed in the favoured odd-A decays, more detailed tests of the present interpretation can be made. These cases are listed in Table III, which gives the observed level spacings in columns four and five.

The ratios of spacings of successive states are seen to agree well with those given by (1). Moreover, as expected, the moments of inertia are similar to those in neighbouring even-even nuclei.

<sup>1</sup> In a quantitative comparison between odd-A and even-even transition probabilities for these  $\Delta I = 0$  favoured transitions, a small correction, of the order of 20 per cent, should be made for the  $l = 2$  contribution to the odd-A decay (cf. (5)).

TABLE II. *Favoured  $\alpha$ -transitions with  $\Delta I = 0$ .*

Parent element	Even-even isotopes ground state transitions			Odd-A isotopes $\Delta I = 0$ favoured transitions		
	A	E (MeV)	F	A	E (MeV)	F
$_{88}\text{Ra}$	220	7.43	0.6	219	8.0	0.3
	222	6.51	1.3	221	6.71	0.3
	224	5.681	1.0	223	5.596	0.2
	226	4.779	0.8	..	..	..
$_{89}\text{Ac}$	..	..	..	223	6.64	0.3
	..	..	..	225	5.80	0.3
	..	..	..	227	4.942	0.2
$_{90}\text{Th}$	224	7.13	1.3	223	7.55	0.5
	226	6.30	1.3	225	6.57	0.4
	228	5.421	0.8	227	5.704	0.2
	230	4.682	0.6	229	4.85	0.5
	232	3.98	1.6	..	..	..
$_{91}\text{Pa}$	..	..	..	227	6.46	0.6
	..	..	..	231	4.722	0.4
$_{92}\text{U}$	228	6.67	0.8	227	6.8	2.0
	230	5.85	1.3	229	6.42	0.3
	232	5.318	0.6	233	4.823	0.5
	234	4.763	0.8	235	4.20	0.3
	236	4.50	1.0	..	..	..
	238	4.18	1.0	..	..	..
$_{93}\text{Np}$	..	..	..	231	6.28	25.1
	..	..	..	233	5.53	2.0
	..	..	..	235	5.06	0.5
	..	..	..	237	4.77	0.4
$_{94}\text{Pu}$	232	6.58	0.1	235	5.85	0.4
	234	6.19	0.6	239	5.150	0.3
	236	5.75	1.0	241	4.893	1.3
	238	5.495	0.6	..	..	..
	240	5.162	0.8	..	..	..
	242	4.898	0.8	..	..	..
$_{95}\text{Am}$	..	..	..	237	6.01	0.1
	..	..	..	239	5.75	0.2
	..	..	..	241	5.476	0.6
	..	..	..	243	5.267	0.8

TABLE II. *Favoured  $\alpha$ -transitions with  $\Delta I = 0$  (continued).*

Parent element	Even-even isotopes ground state transitions			Odd-A isotopes $\Delta I = 0$ favoured transitions		
	A	E (MeV)	F	A	E (MeV)	F
${}_{96}\text{Cm}$	238	6.50	1.6	241	5.89	0.1
	240	6.26	1.0	243	5.777	0.5
	242	6.110	0.6	245	5.34	0.3
	244	5.798	0.8	..	..	..
${}_{97}\text{Bk}$	..	..	..	243	6.20	0.1
	..	..	..	245	5.90	0.3
${}_{98}\text{Cf}$	244	7.15	0.8	249	5.82	0.3
	246	6.75	0.8	..	..	..
	248	6.26	1.3	..	..	..
	250	6.05	0.6	..	..	..
	252	6.15	1.3	..	..	..

Table II. *Favoured  $\alpha$ -transitions with  $\Delta I = 0$ .*

Most of the investigated odd-A  $\alpha$ -decays contain a single component, or a group of components, having transition probabilities of the order of magnitude (4), characteristic of the even-even ground state transitions. These components are interpreted as the favoured transitions which leave the configuration of the last odd particle unchanged. The most energetic of these favoured transitions is expected to have  $\Delta I = 0$ , and to be very similar to the even-even ground state transitions.

The table lists the  $\alpha$ -energies of these favoured  $\Delta I = 0$  transitions in the even-even and odd-A nuclei, together with the reduced transition probabilities  $F$ , which give the ratio between the observed transition probability for the group in question and that calculated from (4) and Table I. (For odd-Z elements, the appropriate coefficients  $C$  and  $D$  have been obtained by interpolation in Table D).

Included in the table are odd-A nuclei for which  $\alpha$ -groups have been observed with  $F \geq 0.1$ . A more detailed test of the classification of these transitions as of the favoured type can be made in cases where the rotational fine structure has been observed (cf. Tables III and IV).

The empirical data is taken from the review articles by HOLLANDER, PERLMAN, and SEABORG, 1953, and by SEABORG, 1954 and, in addition, from GHIORSO et al., 1954, ( $\text{Cf}^{249}$  and  $\text{Cf}^{250}$ ), and THOMPSON et al., 1954 ( $\text{Cf}^{252}$ ). The table includes only nuclei with  $Z \geq 88$ , which are sufficiently far removed from the closed-shell region around  $\text{Pb}^{208}$  that one may expect the simple features associated with the coupling scheme of strongly deformed nuclei.

The tendency toward somewhat larger moments of inertia for these odd-A nuclei as compared with the even-even neighbours continues a trend previously observed in other regions of the periodic table (BOHR and MOTTELSON, 1954).

The value of  $K$  for the rotational band in the daughter

nucleus, as obtained from the observed rotational energies, is listed in column seven of Table III. As is expected for the favoured  $\alpha$ -transitions, it is seen to agree with the  $K$ -values for the parent nucleus in all the cases where the spin of this nucleus has been measured.

The present interpretation of the favoured  $\alpha$ -decays further

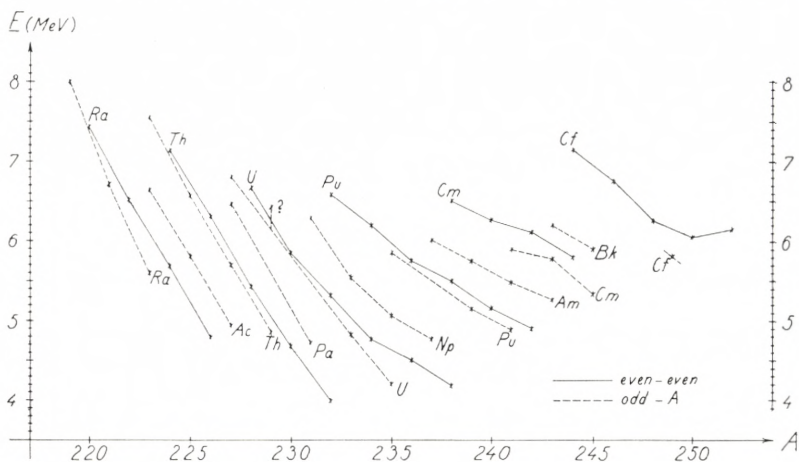


Fig. 4. Energy systematics for favoured  $\alpha$ -transitions. The figure shows the  $\alpha$ -energies for even-even ground state transitions and for the odd-A transitions tentatively classified as favoured and  $\Delta I = 0$  (cf. Table II). As expected, the two types of transitions exhibit closely parallel trends, with the odd-A energies systematically smaller by about 200 keV than those obtained by interpolation between the even-even energies. The most conspicuous deviation is that of  $U^{229}$ , where the classification of the listed group as favoured is rather uncertain, due to the uncertainty in the intensity of the electron capture branch of this decay.

implies that the ground state rotational band in the parent nucleus should have energy spacings very similar to that populated in the daughter. Thus, it is of interest, for example, that in the  $\beta$ -decay of  $Np^{239}$ , leading to  $Pu^{239}$ ,  $\gamma$ -rays are observed (cf. HOLLANDER, PERLMAN, and SEABORG, 1953) with energies 13 and 49 keV, which are very close to the fine structure separations observed in the  $\alpha$ -decay of  $Pu^{239}$  (cf. Table III). In addition, the suggestion (ASARO and PERLMAN, 1952) that the predominant mode of the  $\alpha$ -decay of  $Pu^{231}$  ( $I = 1/2$ ; cf. Table III) does not lead to the ground state of  $U^{235}$  ( $I = 5/2$ ; STUKENBROEKER and McNALLY Jr., 1950) is in agreement with the present interpretation.

The similarity between the favoured transitions in even-even and odd-A nuclei makes possible an estimate of the relative

TABLE III.

Parent nucleus	$I_i = K_i$	$E_0$	$E_1 - E_0$	$E_2 - E_0$	$E_3 - E_0$	$K_f$	$\left(\frac{3\hbar^2}{\mathfrak{I}}\right)$	$\left(\frac{3\hbar^2}{\mathfrak{I}}\right)_{e.e.}$	References
${}_{96}\text{Cm}^{243}$	..	278	44.2	102 (101)	(170)	5/2	38	44	a)
${}_{95}\text{Am}^{243}$	5/2	75	43	97 (98)	(166)	5/2	37	45	b) c)
${}_{95}\text{Am}^{241}$	5/2	60	43.3	98.9 (99.0)	(167)	5/2	37	45	b) d)
${}_{94}\text{Pu}^{239}$	1/2	?	13.5	52.0	(83)	1/2	37	43	e)
${}_{92}\text{U}^{233}$	5/2	0	43	99 (98)	(166)	5/2	37	58	f)
${}_{91}\text{Pa}^{231}$	3/2	331	68	..	..	?	..	68	..

a) F. ASARO (private communication)

b) ASARO and PERLMAN (1954).

c) CONWAY and McLAUGHLIN (1954).

d) MILSTED, ROSENBLUM, and VALADARES (1954).

e) VAN DEN BERG, KLINKENBERG, and REGNAUT (1954).

f) VAN DER SLUIS and McNALLY (1954).

Table III. *Rotational fine structure in favoured odd-A  $\alpha$ -transitions.*

The table collects the data on the rotational bands populated by the favoured  $\alpha$ -decays of odd-A elements. The empirical evidence is taken from the review article by HOLLANDER, PERLMAN, and SEABORG, 1953, except where otherwise noted.

Column two lists the spin of the parent nucleus, where measured. Column three gives the excitation energy  $E_0$  of the state in the daughter, which corresponds to the ground state of the parent, and is populated by the most energetic of the favoured  $\alpha$ -transitions (cf. Table II). The rotational excitations of this state are populated with regularly decreasing intensities and the observed energy spacings  $E_1 - E_0$  and  $E_2 - E_0$  are listed in columns four and five. These states have the spins  $K_f + 1$  and  $K_f + 2$  (cf. (3)), where  $K_f$  is the spin of the  $E_0$  level. The value of  $K_f$  can be determined by means of (1) from the observed energy ratio  $E_2 - E_0$ :  $E_1 - E_0$ , and is listed in column seven. It is seen that, as expected, the values of  $K_f$  are the same as those of  $K_i$  (column two). The energies  $E_2 - E_0$  and  $E_3 - E_0$ , calculated from (1) by adjusting the moment of inertia to the observed value of  $E_1 - E_0$ , are given in parenthesis in columns five and six. The last columns, eight and nine, provide a comparison between the observed moments of inertia for these rotational series and those of the ground state band in the neighbouring even-even nucleus, obtained by removing the last odd nucleon. The irregular fine structure intervals observed for the favoured  $\alpha$ -decays of Pu<sup>239</sup> suggests  $K_f = 1/2$ , in agreement with the measured value  $K_i = 1/2$ ; the decoupling parameter deduced from the observed energies is  $a = -0.26$  (cf. (1)).

†.‡

intensities in the fine structure pattern in odd-A nuclei on the basis of those observed in even-even elements. The principal difference between the two cases is that, in an even-even nucleus, an  $\alpha$ -particle with a given angular momentum,  $l$ , can populate only a single member of a rotational family in the daughter while, for odd-A nuclei, several such states can be populated (except for  $l = 0$ ). Thus, the total emission probability for

$\alpha$ -particles of a given  $l$  in odd-A nuclei is shared among several fine structure components in each rotational band. A simple comparison of the relative intensity patterns for even-even and odd-A nuclei can be obtained in the approximation, discussed above, in which the nucleus may be considered as fixed in space during the traversal of the  $\alpha$ -particle through the non-spherical part of the Coulomb barrier. In this case, the total emission probability for a given  $l$  is the same for the favoured transitions in the even-even and odd-A nuclei, assuming the nuclear deformation to be approximately the same in the two cases. Moreover, the probability that an  $\alpha$ -particle of given  $l$  leaves the daughter in a particular state of the rotational band is given in terms of a vector addition coefficient<sup>1</sup>.

Thus, for a favoured  $\alpha$ -transition from a parent in a state  $I_i, K_i$  to a daughter in a state  $I_f, K_f = K_i$ , one obtains approximately

$$P = P_0(Z, E) \sum_l c_l \langle I_i l K_i 0 | I_f l I_f K_f \rangle^2 \quad (5)$$

for the transition probability per second<sup>2</sup>. The quantity  $P_0$  is given by (4) and is a function of  $Z$  and of the energy  $E$  of the fine structure component in question. The coefficient  $c_l$  which determines the total reduced transition probability for a given  $l$  may be obtained from the observed intensities in the even-even nuclei. The value of  $c_0$  is unity by the definition of  $P_0$ ; the coefficient  $c_2$  is relatively constant for the even-even  $\alpha$ -emitters with  $A > 220$  and is on the average about 0.7<sup>3</sup>; the values of  $c_4$  vary rather strongly with  $A$ , becoming quite small at the very heaviest elements; values of less than 0.01 having been reported (ASARO and PERLMAN, 1953). Only even values of  $l$  are involved, since favoured  $\alpha$ -transitions leave the nuclear parity unaltered. The final factor in (5) is the vector addition coefficient for the addition of the angular momenta  $I_i$  and  $l$  to form the resultant  $I_f$ .

In Table IV, the relative fine structure intensities in the favoured odd-A  $\alpha$ -transitions are compared with those calculated

<sup>1</sup> Similar relative intensity rules apply to  $\beta$ - or  $\gamma$ -transitions of a given multipole order, populating different members of a rotational band (ALAGA, ALDER, BOHR, and MOTTELSON, 1955).

<sup>2</sup> In (5), the small energy dependence of the centrifugal barrier effect is neglected.

<sup>3</sup> Cf. p. 18 for note added in proof.



TABLE IV.

Parent nucleus	$I_i$	Obs. rel. intensities			Calc. rel. intensities				Assumed $c_4$
		$I = K_f, K_f + 1, K_f + 2$			$I = K_f, K_f + 1, K_f + 2, K_f + 3$				
${}_{96}\text{Cm}^{243}$	(5/2)	100	16	~ 4	100	15	2.2	0.03	0.01
${}_{95}\text{Am}^{243}$	5/2	100	15	~ 3	100	14	2.1	0.03	0.01
${}_{95}\text{Am}^{241}$	5/2	100	17	1.7	100	14	2.2	0.03	0.01
${}_{94}\text{Pu}^{239}$	1/2	100	29	16	100	23	19	0.4	0.03
${}_{92}\text{U}^{233}$	5/2	100	18	2.4	100	13	1.8	0.2	0.1
${}_{91}\text{Pa}^{231}$	3/2	100	~ 20		100	11			

Table IV. *Relative intensities of fine structure components in favoured odd-A  $\alpha$ -transitions.*

The empirical data is taken from the references given in Table III. The spins of the parent nuclei are listed in column two. (For  $\text{Cm}^{243}$ , the  $I_i$ -value is deduced from the data in Table III). The observed relative intensities of the favoured  $\alpha$ -transitions leading to the states with spins  $I_f = K_f, K_f + 1, K_f + 2$  are listed in column three. In column four are given the relative intensities calculated from (5) and (4). The coefficient  $c_2$  has been taken to be 0.7, which represents an average value of those measured for the  $l = 2$  transitions in even-even nuclei in this region of elements. The assumed values of  $c_4$ , which are listed in the last column and which only appreciably affect the calculated  $I = K_f + 3$  intensities, are taken from  $l = 4$  transitions in neighbouring even-even nuclei.

from (5). The rather good agreement lends some further support to the present interpretation of these transitions.

The published data test the expression (5) only for transitions involving  $l = 0$  and 2; due to the smallness of  $c_4$  as observed in even-even nuclei, one expects in odd-A nuclei weak transitions populating the higher members of the rotational band in the daughter (cf. Table IV). Thus, for the  $l = 4$  transition from  $\text{Am}^{241}$  to the expected 11/2 state at 226 keV in  $\text{Np}^{237}$  (cf. Fig. 3), one estimates, using  $c_4 = 0.01$  as determined from the  $\text{Pu}^{238}$  decay, an intensity of about .03 %. Recently, an  $\alpha$ -group of the corresponding energy and with an intensity of this order of magnitude has been tentatively reported (ROSENBLUM and VALADARES, private communication).

Expression (5) can also be used for the unfavoured  $\alpha$ -decays, but its application is in such cases more restricted, since the coefficients  $c_l$  are unknown<sup>1</sup>. It may, however, be used to estimate

<sup>1</sup> In unfavoured transitions with  $l \geq K_i + K_f (K_i \neq 0; K_f \neq 0)$ , the expression for  $P$  may contain extra terms, similar to the decoupling term in (1), associated with the symmetrization of the wave function describing the nuclear rotational motion.

relative transition probabilities to different members of a rotational series if only a single value of  $l$  enters significantly (for an example, cf. Fig. 3).

A further test of (5) can be provided by  $\alpha$ - $\gamma$  correlations which may be employed in the case of  $\alpha$ -transitions of mixed  $l$  to yield the relative intensity of the different  $l$ -components. (For an example, cf. Fig. 3).

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*Note added in proof (cf. p. 16):*

Recent studies of the  $\alpha$  fine structure of  $\text{Cf}^{250}$  and  $\text{Cf}^{252}$  (L. B. MAGNUSSON et al., Phys. Rev. **96**, 1576, 1954) and of  $100^{254}$  (F. ASARO, F. STEPHENS, and I. PERLMAN, Bul. Am. Phys. Soc. **29**, no. 8, G4) indicate somewhat smaller values for  $c_2$  ( $\sim 0.25$ ) and larger values for  $c_4$  ( $\sim 0.03$ ) for these new isotopes.

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